EDO-EP-15

September, 1997

Space-time Uncertainty Principle from Breakdown of Topological Symmetry

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Abstract

Starting from topological quantum field theory, we derive space-time uncertainty relation with respect to the time interval and the spatial length proposed by Yoneya through breakdown of topological symmetry in the large N matrix model. This work suggests that the topological symmetry might be an underlying higher symmetry behind the space-time uncertainty principle of string theory.

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1 Introduction

In spite of recent remarkable progress in nonperturbative formulations of M-theory [1] and IIB superstring [2, 3], we have not yet reached a complete understanding of the fundamental principle and the underlying symmetry behind string theory. Since a string has an infinite number of states in the perturbative level in addition to various extended objects as solitonic excitations in the nonperturbative regime, it is expected that in string theory the fundamental principle might have peculiar properties and the gauge symmetry would be quite huge compared to the usual ones in particle theory.

In a quest of the fundamental principle of string theory, Yoneya has advocated a space-time uncertainty principle which is of the form [4, 5]

$$\Delta T \Delta X \geq l_s^2, \tag{1}$$

where l_s denotes the string minimum length which is related to the Regge slope α' by $l_s = \sqrt{\alpha'}$. Provided that the relation (1) holds literally, string theory would lead a physical picture that space-time in itself is quantized at the short distance and the concept of space-time as a continuum manifold cannot be extrapolated beyond the fundamental string scale l_s. In more recent work [5], making use of the "conformal constraint" which stems from the Schild action [6] and essentially expresses the space-time uncertainty priciple (1), Yoneya has constructed a IIB matrix model from which the IKKT model [2] can be interpreted as an effective theory for D-branes [7].

In this article, we would like to consider the space-time uncertainty principle from a different perspective from Yoneya's one. Namely, we attempt to understand at least some aspects of the underlying gauge symmetry of string theory through the relation (1). We will see that topological quantum field theory [8] provides us with a nice framework in understanding the space-time uncertainty principle (1). It is worthwhile to point out that it has been already stated that a topological symmetry might be of critical importance in both string theory and quantum gravity in connection with the background independent formulation of string theory and the unbroken phase of quantum gravity [9]. Our study at hand may shed a light on this idea to some extent.

The paper is organized as follows. In section 2 we review Yoneya's works [5] relating to the present study. In section 3, we derive the space-time uncertainty principle from the topological field theory where the classical action is trivially zero. The final section is devoted to discussions.

2 The space-time uncertainty principle and the conformal constraint

In this section, we review only a part of Yoneya's works relevant to later study (See [4, 5] for more detail). Let us start with the Schild action [6] of a bosonic string. Then the Schild

action has a form

$$S_{Schild} = -\frac{1}{2} \int d^2\xi \left[-\frac{1}{2\lambda^2} \frac{1}{e} \left(\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu \right)^2 + e \right], \tag{2}$$

where $X^\mu(\xi)$ $(\mu = 0, 1, \dots, D-1)$ are space-time coordinates, $e(\xi)$ is a positive definite scalar density defined on the string world sheet parametrized by ξ^1 and ξ^2, and $\lambda = 4\pi\alpha'$.

Taking the variation with respect to the auxiliary field $e(\xi)$, one obtains

$$e(\xi) = \frac{1}{\lambda} \sqrt{-\frac{1}{2} \left(\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu \right)^2}, \tag{3}$$

which is also rewritten to be

$$\lambda^2 = -\frac{1}{2} \left\{ X^\mu, X^\nu \right\}^2, \tag{4}$$

where one has introduced the diffeomorphism invariant Poisson bracket defined as

$$\{X^\mu, X^\nu\} = \frac{1}{e(\xi)} \varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu. \tag{5}$$

Then eliminating the auxiliary field $e(\xi)$ through (3) from (2) and using the identity

$$-\det \partial_a X \cdot \partial_b X = -\frac{1}{2} \left(\varepsilon^{ab} \partial_a X^\mu \partial_b X^\nu \right)^2, \tag{6}$$

the Schild action (2) becomes at least classically equivalent to the famous Nambu-Goto action S_{NG}

$$\begin{aligned} S_{Schild} &= -\frac{1}{\lambda} \int d^2\xi \sqrt{-\det \partial_a X \cdot \partial_b X}, \\ &= S_{NG}. \end{aligned} \tag{7}$$

Let us note that the "conformal" constraint (4) describes half the classical Virasoro conditions [5] and the well-known relation between the Poisson bracket and the commutation relation in the large N matrix model

$$\{A, B\} \longleftrightarrow [A, B], \tag{8}$$

leads the "conformal" constraint (4) to

$$\lambda^2 = -\frac{1}{2} \left[X^\mu, X^\nu \right]^2. \tag{9}$$

Then it turns out that the commutation relation (9) yields the space-time uncertainty principle (1) [5]. Here notice that it is not the whole Schild action but the "conformal" constraint in the large N matrix model that produces the space-time uncertainty principle so that it is a natural next step to seek the fundamental action yielding the relation (9). Actually, Yoneya

has derived such an action which has a close connection with the IKKT model [5]. His construction of the action is in itself quite interesting but seem to be a bit ambiguous. In particular, a natural question arises whether or not we can derive the relation (9) in terms of the more field-theoretic framework where we usually start with a classical action with some local symmetry. In the following section, we shall challenge this problem of which we will see an interesting possibility that the breakdown of a topological symmetry gives a generation of the quantum action including the essential content of the space-time uncertainty principle.

3 A topological model

Let us start by considering a topological theory [8] where the classical action is trivially zero but dependent on the fields $X^\mu(\xi)$ and $e(\xi)$ as follows:

$$S_c = S_c(X^\mu(\xi), e(\xi)) = 0. \tag{10}$$

The BRST transformations corresponding to the topological symmetry are given by

$$\begin{aligned} \delta_B X^\mu &= \psi^\mu, \ \ \delta_B \psi^\mu = 0, \\ \delta_B e &= e\ \eta, \ \ \delta_B \eta = 0, \\ \delta_B \bar{c} &= b, \ \ \delta_B b = 0, \end{aligned} \tag{11}$$

where ψ^μ and η are ghosts, and $\bar{c}$ and b are respectively an antighost and an auxiliary field. Note that these BRST transformations are obviously nilpotent. Also notice that the BRST transformation $\delta_B e$ shows the character as a scalar density of e.

The idea, then, is to fix partially the topological symmetry corresponding to $\delta_B e$ by introducing an appropriate covariant gauge condition. A conventional covariant and nonsingular gauge condition would be $e = 1$ but this gauge choice is not suitable for the present purpose since it makes difficult to move to the large N matrix theory. Then almost unique choice up to its polynomial forms is nothing but the "conformal" condition (4). Hence the quantum action defined as $S_q = \int d^2\xi\ e L_q$ becomes

$$\begin{aligned} L_q &= \frac{1}{e}\delta_B \left[\bar{c} \left\{ e \left(\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 \right) \right\} \right], \\ &= b \left(\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 \right) - \bar{c} \left(\eta \left(-\frac{1}{2} \{X^\mu, X^\nu\}^2 + \lambda^2 \right) + 2 \{X^\mu, X^\nu\} \{X^\mu, \psi^\nu\} \right), \end{aligned} \tag{12}$$

where the BRST transformations (11) were used.

What is necessary to obtain a stronger form of the space-time uncertainty relation (9) is to move to the large N matrix theory where in addition to (8) we have the following correspondences

$$\begin{aligned} \int d^2\xi\ e &\longleftrightarrow Trace, \\ \int De &\longleftrightarrow \sum_{n=1}^{\infty}, \end{aligned} \tag{13}$$

where the trace is taken over $SU(n)$ group. These correspondences can be justified by expanding the hermitian matices by $SU(n)$ generators in the large N limit as is reviewed by the reference [10]. Here it is worth commenting one important point. As in the IKKT model [2] the matrix size n is now regarded as a dynamical variable so that the partition function includes the summation over n. Even if the direct proof is missing, the summation over n is expected to recover the path integration over $e(\xi)$. In fact, the authors of the reference [3] have recently shown that the model of Fayyazuddin et al. [10] where a positive definite hermitian matrix Y is introduced as a dynamical variable instead of n, belongs to the same universality class as the IKKT model [2] owing to irrelevant deformations of the loop equation [3]. Thus we think that the correspondences (8) and (13) are legitimate even in the context at hand.

Now in the large N limit, we have

$$S_q = Tr\left(b\left(\frac{1}{2}\left[X^\mu, X^\nu\right]^2 + \lambda^2\right) - \bar{c}\left\{\eta\left(-\frac{1}{2}\left[X^\mu, X^\nu\right]^2 + \lambda^2\right) + 2\left[X^\mu, X^\nu\right]\left[X^\mu, \psi^\nu\right]\right\}\right). \quad (14)$$

Next by redefining the auxiliary field b by $b + \bar{c}\,\eta$, S_q can be cast into a simpler form

$$S_q = Tr\left(b\left(\frac{1}{2}\left[X^\mu, X^\nu\right]^2 + \lambda^2\right) - 2\lambda^2\bar{c}\,\eta - 2\bar{c}\left[X^\mu, X^\nu\right]\left[X^\mu, \psi^\nu\right]\right). \quad (15)$$

Then the partition function is defined as

$$\begin{aligned} Z &= \int DX^\mu D\psi^\mu De D\eta D\bar{c} Db\; e^{-S_q}, \\ &= \sum_{n=1}^{\infty} \int DX^\mu D\psi^\mu D\eta D\bar{c} Db\; e^{-S_q}. \end{aligned} \quad (16)$$

At this stage, it is straightforward to perform the path integration over η and $\bar{c}$, as a result of which one obtains

$$Z = \sum_{n=1}^{\infty} \int DX^\mu D\psi^\mu Db\; e^{-Tr\; b\; \left(\frac{1}{2}[X^\mu, X^\nu]^2 + \lambda^2\right)}. \quad (17)$$

In (17) there remains the gauge symmetry

$$\delta\psi^\mu = \omega^\mu, \quad (18)$$

which is of course the remaining topological symmetry. Now let us factor out this gauge volume or equivalently fix this gauge symmetry by the gauge condition $\psi^\mu = 0$, so that the partition function is finally given by

$$Z = \sum_{n=1}^{\infty} \int DX^\mu Db\; e^{-Tr\; b\; \left(\frac{1}{2}[X^\mu, X^\nu]^2 + \lambda^2\right)}. \quad (19)$$

It is remarkable that the variation with respect to the auxiliary variable b in (19) gives a stronger form of the space-time uncertainty relation (9) and the theory is "dynamical" in

the sense that the ghosts have completely decoupled from (19). In other words, we have shown how to derive the space-time uncertainty principle from a topological theory through the breakdown of a topological symmetry in the large N matrix model. Why has the topological theory yielded the nontrivial "dynamical" theory? The reason is very much simple. In moving from the continuous theory (12) to the matrix theory (14), the dynamical degree of freedom associated with $e(\xi)$ was replaced by the discrete sum over n, on the other hand, the corresponding BRST partner η remains the continuous variable. This distinct treatment of the BRST doublet leads to the breakdown of the topological symmetry giving rise to a "dynamical" matrix theory. In this respect, it is worthwhile to point out that while the topological symmetry is "spontaneously" broken, the other gauge symmetries never be violated in the matrix model (Of course, correctly speaking, these gauge symmetries reduce to the global symmetries in the matrix model but this is irrelevant to the present argument.) Moreover, notice that the above-examined phenomenon is a peculiar feature in the matrix model with the scalar density $e(\xi)$, which means that an existence of the gravitational degree of freedom is an essential ingredient.

4 Discussions

In this short article, we have investigated a possibility of the space-time uncertainty principle advocated by Yoneya [4, 5] to be derived from the topological field theory [8]. The study at hand suggests that the underlying symmetry behind this principle in string theory might be a topological symmetry as mentioned before in a different context [9]. This rather unexpected appearance of the topological field theory seems to be plausible from the following arguments. Suppose that we live in the world where the topological symmetry is exactly valid. Then we have no means of measuring the distance owing to lack of the metric tensor field so that there is neither concept of distance nor the space-time uncertainty principle. If the topological symmetry, in particular, that associated with the gravitational field, is spontaneously broken by some dynamical mechanism, an existence of the dynamical metric together with a string would give us both concept of distance and the space-time uncertainty principle.

So far we have not paid attention to the number of the space-time dimensions so much except the implicit assumption $D \geq 2$. An intriguing case is $D = 2$ even if this specification is not always necessary within the present formulation. In this special dimension, the Nambu-Goto action which is at least classically equivalent to the Schild action as shown in (7) becomes not only the topological field theory but also almost a surface term as follows:

$$\begin{aligned} \sqrt{-\det \partial_a X \cdot \partial_b X} &= \sqrt{-\left(\det \partial_a X^\mu\right)^2}, \\ &= \pm \det \partial_a X^\mu, \\ &= \mp \frac{1}{2} \varepsilon^{ab} \varepsilon_{\mu\nu} \partial_a X^\mu \partial_b X^\nu, \end{aligned} \tag{20}$$

where we have assumed a smooth parametrization of X^μ over ξ^a in order to take out the absolute value. Actually, this topological model has been investigated to some extent in the

past [11, 12, 13]. In this case, it is interesting that we can start with the nonvanishing surface term as a classical action.

One of the most important problems in future is to understand the symmetry breaking mechanism of a topological symmetry proposed in this paper more clearly by physical picture. Another interesting problem is to introduce the spinors and construct a supersymmetric matrix model from the topological field theory. These problems will be reported in a separate publication.

Acknowledgement

The author thanks Y.Kitazawa and A.Sugamoto for valuable discussions. He is also indebted to M.Tonin for stimulating discussions and a kind hospitality at Padova University where most of parts of this study have been done. This work was supported in part by Grant-Aid for Scientific Research from Ministry of Education, Science and Culture No.09740212.

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